MATHEMATICS FOR FINANCE Exam

Date ________________

Surname ____________________________ **Name** ____________________________

ID Number ______________________

Exercise 1. Given the canonical basis $\mathcal{B}_{\mathbb{R}^3} = \{\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3\}$ and $\mathcal{B}_{\mathbb{R}^4} = \{\boldsymbol{f}_1, \boldsymbol{f}_2, \boldsymbol{f}_3, \boldsymbol{f}_4\}$ of the vector spaces $\mathbb{R}^3, \mathbb{R}^4$, respectively, and the linear application $L : \mathbb{R}^3 \longrightarrow \mathbb{R}^4$ acting on the basis vectors of $\mathbb{R}^3$ according the transformation laws

$$\begin{cases} L(\boldsymbol{e}_1) = \text{linear combination of } \boldsymbol{f}_1, \boldsymbol{f}_2, \boldsymbol{f}_3, \boldsymbol{f}_4 \\ L(\boldsymbol{e}_2) = \text{linear combination of } \boldsymbol{f}_1, \boldsymbol{f}_2, \boldsymbol{f}_3, \boldsymbol{f}_4 \\ L(\boldsymbol{e}_3) = \text{linear combination of } \boldsymbol{f}_1, \boldsymbol{f}_2, \boldsymbol{f}_3, \boldsymbol{f}_4 \,, \end{cases}$$

1) write the matrix A associated to the linear application L with respect to the given basis;

2) find the subspaces *kernel* and *image* the linear application L determining a basis for both subspaces.

Let us consider the linear application $\tilde{L} : \mathbb{R}^4 \longrightarrow \mathbb{R}^3$ defined by the transformation laws of the components

$$(y_1, y_2, y_3) = \tilde{L}(x_1, x_2, x_3, x_4)$$

with

$$y_1 = \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 + \alpha_4 x_4 \,, \qquad y_2 = \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 \,,$$
$$y_3 = \gamma_1 x_1 + \gamma_2 x_2 + \gamma_3 x_3 + \gamma_4 x_4 \,,$$

where in the vector spaces $\mathbb{R}^4, \mathbb{R}^3$ the same basis $\mathcal{B}_{\mathbb{R}^4}, \mathcal{B}_{\mathbb{R}^3}$ as before are fixed.

3) Write the matrix B associated to the linear application $\tilde{L}$ with respect to the given basis and determine the matrix, denoted by M, associated to the product of linear applications in the order $\tilde{L}L$.

4) Verify whether the matrix M is diagonalizable by taking into account that the vector $\boldsymbol{v} = (a, b, c)$ is an eigenvector of M.

If M is diagonalizable,

5) find the basis vectors with respect to which the matrix M assumes a diagonal form denoted by $\mathcal{D}$ and write the matrix C of the basis change such that $C^{-1}MC = \mathcal{D}$;

6) write the diagonal matrix $\mathcal{D}$ (without performing the matrix multiplication $C^{-1}MC$).

Exercise 2. Solve the following Cauchy problem

$$\begin{cases} ay'''(x) + by''(x) + cy'(x) + dy(x) = f(x) \\ y(0) = y_0 \\ y'(0) = y_1 \\ y''(0) = y_2 \end{cases}$$

Exercise 3. Find the optimal points of the function

$$w = f(x, y, z)$$

subject to the constraints

$$\begin{cases} g_1(x, y, z) = 0 \\ g_2(x, y, z) = 0 \end{cases}$$