Quantitative Portfolio Selection for Management

Academic Year 2025/2026

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1. **Exercise 1**

   Let us consider a market with two assets a_1 and a_2 with expected returns equal to $\mu_1 = 5,62\%$ and $\mu_2 = 7.32\%$, and standard deviations equal to $\sigma_1 = 11,38\%$ and $\sigma_2 = 15,23\%$ respectively. Moreover let their correlation be $\rho = -0.27$. Consider the portfolio P with weights $w_1 = 26\%$ and $w_2 = 74\%$. Compute:

   (a) the expected return of P

   - ◯ 6,53%
   - ◯ 6,31%
   - ◯ 6,72%
   - ◯ 6,88%

   (b) the standard deviation of P

   - ◯ 10,63%
   - ◯ 10,85%
   - ◯ 10,47%
   - ◯ 10,98%

   (c) the weight composition of the minimum variance portfolio

   - ◯ $w_1 = 61,53\%, \quad w_2 = 38,47\%$
   - ◯ $w_1 = 61,26\%, \quad w_2 = 38,74\%$
   - ◯ $w_1 = 61,38\%, \quad w_2 = 38,62\%$
   - ◯ $w_1 = 61,42\%, \quad w_2 = 38,58\%$

   (d) the expected return of the minimum variance portfolio

   - ◯ 6,05%
   - ◯ 6,43%
   - ◯ 6,28%
   - ◯ 6,17%

   (e) the standard deviation of return of the minimum variance portfolio

   - ◯ 7,82%
   - ◯ 7,49%
   - ◯ 7,95%
   - ◯ 7,63%

2. **Exercise 2**

Let us consider the equally weighted portfolio EW, that is $w = (25\%, 25\%, 25\%, 25\%)$, of four assets, a_1, a_2, a_3, a_4, whose vector of expected returns and covariance matrix are given by

$$\mu = \begin{pmatrix} 6,27\% \\ 8,37\% \\ 7,43\% \\ 4,85\% \end{pmatrix} \qquad \text{and} \qquad C = \begin{pmatrix} 0,0028 & 0,0011 & 0,0008 & 0,0012 \\ 0,0011 & 0,0027 & -0,0007 & 0,0017 \\ 0,0008 & -0,0007 & 0,0035 & 0,0013 \\ 0,0012 & 0,0017 & 0,0013 & 0,0021 \end{pmatrix}$$

respectively. Compute

(a) the expected return of EW

- ◯ 6,89%
- ◯ 6,73%
- ◯ 6,35%
- ◯ 6,58%

(b) the standard deviation of returns of EW

- ◯ 3,62%
- ◯ 3,48%
- ◯ 3,70%
- ◯ 3,51%

(c) the correlation of returns of asset a_2 with asset a_4

- ◯ -22,77%
- ◯ 71,39%
- ◯ 25,56%
- ◯ 40,01%

(d) the standard deviation of returns of asset a_2

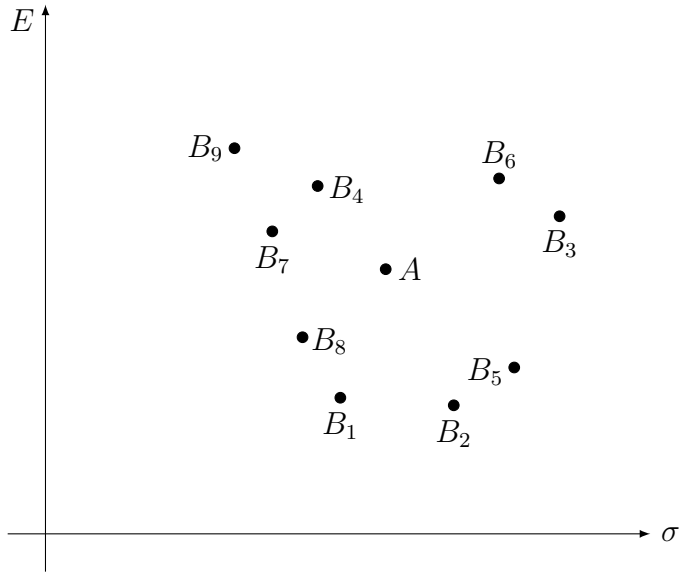
- ◯ 5,20%
- ◯ 5,32%
- ◯ 5,73%
- ◯ 5,57%

3. **Exercise 3** One of the following matrices cannot be a covariance matrix. Identify it and explain why

$$A = \begin{pmatrix} 4 & 3,5 \\ 3,5 & 5 \end{pmatrix} \qquad B = \begin{pmatrix} 3 & 2,7 \\ 2,4 & 4 \end{pmatrix} \qquad C = \begin{pmatrix} 2 & -1 \\ -1 & 5 \end{pmatrix} \qquad D = \begin{pmatrix} 4 & 1,3 \\ 1,3 & 2 \end{pmatrix}$$

Question

Let us consider the following figure where σ, E denote the *standard deviation* and the *expected value* (or *mean value*) of the returns of an *asset*, respectively.



Among the *assets* B_i, with $i = 1, 2, 3, \ldots, 9$,

1) determine the *assets* which are *surely* better than the *asset* A according to the bi-criteria σ, E,

2) determine the *assets* which can be equivalent to the *asset* A according to the bi-criteria σ, E,

3) determine whether there exists the *asset* which is the *global best*, according to the bi-criteria σ, E, and specify it.

Give a motivation for each answer.